Nonlinear Three-dimensional Wind Field Retrieval of Typhoon Based on Modified VAD Analysis

Poster3-11

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1. INTRODUCTION

Three-dimensional (3D) wind field retrieval is important in the numeric weather forecast. The Doppler weather radar commonly used in weather systems can only detect the radial velocity of the precipitation particle along the line of sight of radar. So it needs to develop 3D wind field retrieval algorithms using the radial Doppler velocity data.

In the traditional ground-based radar system, the 3D wind field retrieval m1024ethod represented by the velocity azimuth display (VAD) analysis is to express the radial Doppler velocity as a Fourier series with azimuth angle[1]. Under the assumption of constant or linear horizontal wind distribution, the Fourier coefficients obtained by least-squares fit represent the average wind field information in the VAD circle, and the vertical wind field profile information can be obtained by processing the data of VAD circles with different elevation angles. Currently, VAD technology has been applied to many radar systems[2].

The height of typhoon is about 15km[3], and the Doppler weather radar onboard near space platform is located at a height of about 5 km above the typhoon, which can perform the look-down cone-scanning. However, due to the differences of the ground-based radar, there are some problems to be considered when detecting. Firstly, when the platform is measured close to the typhoon, the beam footprint will be spiral due to the motion of platform. Secondly, for far-range detection requirements, the beam may sweep through the typhoon, eye wall and spiral rain-band in sequence during the detection, where the wind field cannot be assumed to be constant or linear. Finally, for a VAD circle, some singular point caused by the random factors such as platform shake needs to be eliminated, which will cause the lack of measurements.

In order to more accurately reproduce the 3D wind field, the modified VAD analysis in the Doppler radar onboard near space platform scanning geometry mode is proposed. The Doppler velocity considering the platform motion is derived. The measurements including missing data can be disposed by the extend velocity azimuth display (EVAD) analysis[4, 5]. The nonlinear 3D wind field can be retrieved by solving multiple sets of data in one circle. Compared with the traditional VAD analysis, the modified VAD analysis can retrieve the more realistic atmospheric wind field, which have better precision and wider application.

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2. METHODOLOGY

2.1 Traditional VAD analysis

The traditional ground-based radar VAD analysis obtains the wind field information by 360° omnidirectional circumferential scanning at a fixed elevation angle to obtain the radial Doppler velocity V_r as shown in Figure 1 .

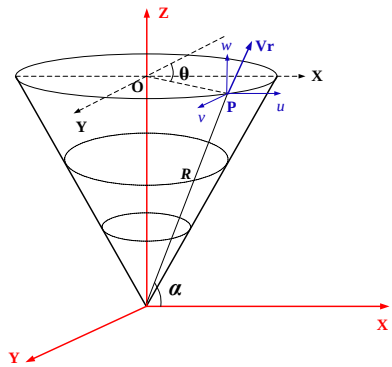

Figure 1 The geometry of traditional VAD method

In Figure 1, the X axis points to east, the Y axis points to north and the Z axis points to zenith. V_r is the radial doppler velocity, θ is azimuthal angle, u,v,w are the wind field pointing to X,Y and Z, respectively. α is elevation angle, R is the range from platform to particle. Assuming that the horizontal wind is linear and the vertical wind is constant, the relationship between V_r and u,v,w can be written as:

$$V_r(\theta) = v\cos\theta\cos\alpha + u\sin\theta\cos\alpha + w\sin\alpha \quad (1)$$

Assuming that the horizontal wind is spatially linear and the vertical wind is constant, u and v can be expanded as first-order Taylor at the scanning center O:

$$\begin{aligned} u &= u_0 + u_x \cdot x + u_y \cdot y \\ v &= v_0 + v_x \cdot x + v_y \cdot y \end{aligned} \quad (2)$$

The coordinate transformation relationship can be expressed as:

$$\begin{aligned} x &= R\cos\alpha\sin\theta \\ y &= R\cos\alpha\cos\theta \\ z &= R\sin\alpha \end{aligned} \quad (3)$$

Substituting (3) into (2) and substituting into (1), the Fourier expansion of the radial Doppler velocity V_r as a function of azimuth angle θ can be obtained:

$$V_r(\theta) = a_0 + a_1\sin\theta + a_2\cos\theta + a_3\cos 2\theta + a_4\sin 2\theta \quad (4)$$

where :

$$
\begin{aligned}
a_0 &= \frac{1}{2}R\cos^2\alpha\left(u_x+v_y\right)+w\sin\alpha \\
a_1 &= v_0\cos\alpha \\
a_2 &= u_0\cos\alpha \\
a_3 &= \frac{1}{2}R\cos^2\alpha\left(v_y-u_x\right) \\
a_4 &= \frac{1}{2}R\cos^2\alpha\left(v_x+u_y\right)
\end{aligned}
\tag{5}
$$

The expressions of horizontal wind field information can be obtained from a_1 and a_2 in (5). The vertical wind velocity w can be obtained according to the mass continuous equation to integrate the divergence:

$$
\begin{aligned}
v_0 &= a_1/\cos\Phi \\
u_0 &= a_2/\cos\Phi \\
w_0 &= -\int_{z_0}^{z}\left(u_x+v_y\right)dz = -\int_{z_0}^{z} div\mathbf{V}dz
\end{aligned}
\tag{6}
$$

2.2 The modified VAD analysis

When the beam is scanned by the cone, the beam footprint will be spiral due to the platform motion. For far-range detection requirements, the beam may sweep through the typhoon, eye wall and spiral rain-band in sequence during the detection, where the wind field cannot be assumed to be constant or linear. Besides, the lack of measurements causes the uniformly-spaced samples in one circle. These will cause the inapplicability of traditional VAD analysis.

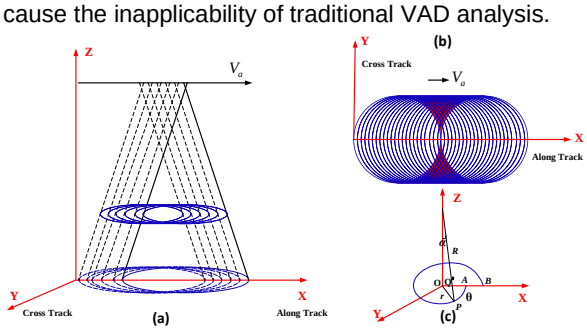

Figure 2 The geometry of modified VAD method

The system geometry of the look-down cone-scanning Doppler radar is shown in Figure 2. V_a is the platform velocity, θ is the azimuthal angle, $-\pi/2-\alpha$ is the elevation angle, R is the range from platform to particle and r is the scanning radius. The radial Doppler velocity V_r can be written as:

$$
V_r\left(\theta\right) = u\sin\theta\cos\alpha + v\cos\theta\cos\alpha - w\sin\alpha \tag{7}
$$

Assuming that the wind field in horizontal is not constant or linear and has second-order distribution, u and v can be expanded as second-order Taylor at the scanning center O:

$$
\begin{aligned}
u &= u_0 + u_x\cdot x + u_y\cdot y + \frac{1}{2}u_{xx}\cdot x^2 + u_{xy}\cdot xy + \frac{1}{2}u_{yy}\cdot y^2 \\
v &= v_0 + v_x\cdot x + v_y\cdot y + \frac{1}{2}v_{xx}\cdot x^2 + v_{xy}\cdot xy + \frac{1}{2}v_{yy}\cdot y^2
\end{aligned}
\tag{8}
$$

The coordinate transformation relationship for spiral trajectory can be expressed as:

$$
\begin{aligned}
x &= R\cos\alpha\sin\theta \\
y &= R\cos\alpha\cos\theta + \left(\frac{V_aT}{2\pi}\right)\theta \\
z &= R\sin\alpha
\end{aligned}
\tag{9}
$$

where T is the period of scanning one circle. The radial doppler velocity V_r can be performed as Fourier series expansion:

$$
V_r\left(\theta\right) = a_0 + \sum_{p=1}^{N}\left(a_p\sin n\theta + b_p\cos n\theta\right) + C \tag{10}
$$

where a_p and b_p is p-order Fourier coefficient, N is the truncation order, C is the item related to platform motion V_a. Due to that u_0 and v_0 exist in a_1 and b_1, substituting (9) into (8) and into (10), when the truncation order is N, a_1 and b_1 can be written as:

$$
\begin{aligned}
a_1 &= u_0\cos\alpha + A_1R^2\cos^3\alpha + \ldots + A_mR^{2m}\cos^{2m+1}\alpha + C_{a1} \\
&= \sum_{i=0}^{m}A_iR^{2i}\cos^{2i+1} + C_{a1} \\
b_1 &= v_0\cos\alpha + B_1R^2\cos^3\alpha + \ldots + B_mR^{2m}\cos^{2m+1}\alpha + C_{b1} \\
&= \sum_{i=0}^{m}B_iR^{2i}\cos^{2i+1} + C_{b1}
\end{aligned}
\tag{11}
$$

$$
\begin{cases}
m = \dfrac{N-1}{2}, N \quad is \quad odd \\
m = \dfrac{N-2}{2}, N \quad is \quad even
\end{cases}
\tag{12}
$$

where $C_{a1} = \frac{V_aT}{2\pi}\theta\cos\alpha\left(u_y + u_{yy}\frac{V_aT}{4\pi}\theta\right)$ and $C_{b1} = \frac{V_aT}{2\pi}\theta\cos\alpha\left(v_y + v_{yy}\frac{V_aT}{4\pi}\theta\right)$ are the items related to platform motion V_a. u_0 and v_0 can be obtained from different Doppler velocity data for the same circle within a short time and fitting them to solve several equations with (11) form.

When there are many missing measurements, the EVAD algorithm for non-omnidirectional sampling can be used to solve the Fourier coefficient[4].

Assuming that:

$$
\begin{aligned}
\mathbf{f}\left(\theta\right) &= \left[f_0\left(\theta\right), f_1\left(\theta\right), f_2\left(\theta\right), f_3\left(\theta\right), f_4\left(\theta\right), \cdots\right] \\
&= \left[1, \sin\theta, \cos\theta, \sin 2\theta, \cos 2\theta, \cdots\right]
\end{aligned}
\tag{13}
$$

$$
\varsigma = [a_0, a_1, b_1, a_2, b_2, \cdots]^T \tag{14}
$$

where $\mathbf{f}\left(\theta\right)$ is basal function, ς is Fourier coefficient, according to the least-squares principle:

$$
\frac{\partial\left|V_r' - V_r\right|}{\partial\varsigma} = 0 \tag{15}
$$

ς can be solved as :

$$
\varsigma = \mathbf{F}^{-1}\boldsymbol{\kappa} \tag{16}
$$

where $\kappa_p = \sum_{j=1}^{M} V_r\left(\theta_j\right) f_p\left(\theta_j\right)$, $\mathbf{F}$ is the covariance matrix of $f_p\left(\theta_j\right)$. V_r and V_r' are the measurements and estimations of Doppler velocity, respectively. M is the sampling number for one circle.

3. SIMULATIONS AND RESULTS

When the wind field is nonlinear, the data has 10% missing area and the input nonlinear wind field is shown Figure 3(a), where the first-order derivative of wind velocity is $10^{-3}s^{-1}$ and the second-order one is 10^{-8} .

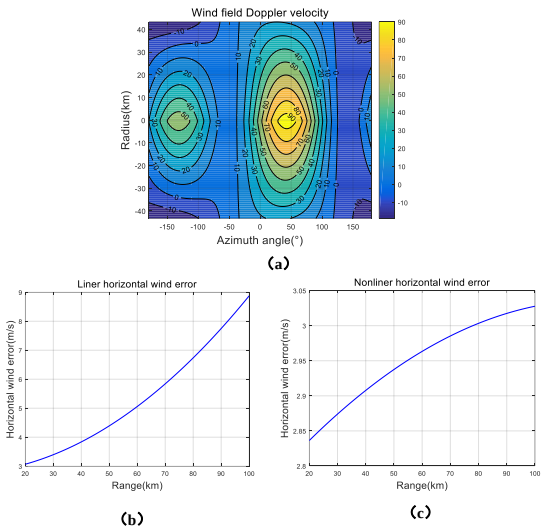

Figure 3 Simulation results.
(a) Input wind field; (b)Linear horizontal wind error; (c)Nonlinear horizontal wind error

The horizontal wind field errors from the traditional linear VAD method are shown in Figure 3 (b). Figure 3 (c) are the horizontal wind field errors from the modified VAD method. It can be found that the modified VAD method has a better retrieval accuracy.

4. CONCLUSIONS

When the wind field changes greatly and the data has a missing measurement area, the traditional VAD technique will cause a large error under the assumption of linear wind field. In the case of linear changes and lack of measurement points, the modified VAD technology applied to the look-down conical-scanning motion platform and nonlinear wind field can be used to obtain the wind field information, and the accuracy is better than the traditional VAD method.

5. REFERENCES

[1] K. A. Browning and R. Wexler, "The Determination of Kinematic Properties of a Wind Field Using Doppler Radar," *Journal of Applied Meteorology,* vol. 7, no. 1, pp. 105-113, 1968.

[2] L. Tian, G. M. Heymsfield, A. C. Didlake, S. Guimond, and L. Li, "Velocity–Azimuth Display Analysis of Doppler Velocity for HIWRAP," *Journal of Applied Meteorology & Climatology,* vol. 54, p. 150713095729000, 2014.

[3] S. Raghavan, *Radar Meteorology*. 2003.

[4] T. Matejka and R. C. Srivastava, "An improved version of the extended velocity-azimuth display analysis of single-Doppler radar data," *Journal of Atmospheric & Oceanic Technology,* vol. 8, no. 4, pp. 453-466, 1991.

[5] R. Rabin and D. Zrnic, "Subsynoptic-Scale Vertical Wind Revealed by Dual Doppler-Radar and VAD Analysis," *Journal of the Atmospheric Sciences,* vol. 37, no. 3, pp. 644-654, 1980.